## CSCI 2824 - Discrete Structures Test 2 Review

- 1. For the following determine if A = B
  - (a)  $C = \{1, 2, 3\}, D = \{2, 3, 4\}, A = \{2, 3\}, B = C \cap D.$
  - (b)  $A = \{1, 2, 3\}, B = \{n : n \in \mathbb{Z}^+ \text{ and } n^2 < 10\}$
  - (c)  $A = \{1, 3, 5\}, B = \{n : n \in \mathbb{Z}^+ \text{ and } n^2 1 < n\}$
  - (d)  $A = \{x : x \in \mathbb{R} \text{ and } 0 \le x \le 2\}, B = \{1, 2\}.$
- 2. For the following questions use represent the propositions using the given symbols, and determine whether the proposition is true or not.

$$p: 5 < 9$$
  
 $q: 9 < 7$   
 $r: 5 < 7$ 

- (a) 5 < 9 and 9 < 7
- (b) It is not the case that (5 < 9 and 9 < 7)
- 3. For the following questions assume that p and r are false and that q and s are true. Determine the truth value of the given proposition.
  - (a)  $(p \to r) \land (q \to r)$
  - (b)  $p \to (q \to r)$
  - (c)  $(s \to (p \land \neg r)) \land ((p \to (r \lor q)) \land s)$
  - (d)  $((p \land \neg q) \to (q \land r)) \to (s \lor \neg q)$
- 4. Prove that for all integers m and n, if m and n are even then mn is even.
- 5. Prove that for every rational number x if  $x \neq 0$  then 1/x is rational.
- 6. Prove that if  $X \subseteq Y$  then  $Y \setminus (Y \setminus X) = X$  for all sets X and Y.
- 7. Prove that  $m^3 + 2n^2 = 36$  has no solution in the positive integers.
- 8. By experimenting with small values of n, guess a formula for the given sum:

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \dots + \frac{1}{n(n+1)}$$

then use induction to verify your formula.

- 9. Determine whether the following functions are one-to-one or onto or both or neither. Each function is of the form  $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ 
  - (a) f(m,n) = m
  - (b)  $f(m,n) = m^2 + n^2$
  - (c) f(m,n) = m + n + 2
- 10. Let X be a non-empty set. Define a relation on  $\mathcal{P}(X)$ , the powerset of X, as  $(A, B) \in R$  if  $A \subseteq B$ . Is this relation reflexive, symmetric, anitymmetric, transitive, and/or a partial order?
- 11. Show that:

- (a)  $n! \in O(n^n)$
- (b)  $2^n \in O(n!)$
- 12. Add the following numbers (without changing to base-10):
  - (a)  $4A_{16} + B4_{16}$
  - (b)  $82054_{16} + AEFA3_{16}$
  - (c)  $1001_2 + 1111_2$
  - (d)  $1101_{16} + 101100_2 + 11011011_2$
- 13. In the following exercises a six person committee composed of Alice, Ben, Connie, Dolph, Ebert and Francisco is to select a chairperson, secretary and treasurer among themselves.
  - (a) How many selections exclude Connie?
  - (b) How many selections are there in which both Ben and Francisco are officers?
  - (c) How many selections are there in which Dolph is either chairperson or he is not an officer?
- 14. In how many ways can five distinct Martians, ten distinct Vesuvians and eight distinct Jovians wait in line if no two Martians stand together.
- 15. In how many ways can five distinct Martians and five distinct Jovians be seated at a circular table if no two Martians sit together. (Remember that location doesn't matter, but who you sit next to does.)
- 16. In how many ways can 15 identical math books be distributed among 6 students?
- 17. Is this graph bipartite?



18. For the following graph find a path with no repeated edges from d to e containing all edges. (Or explain why it doesn't exist)

