# CSCI 2824 - Discrete Structures
# Test 1 Review

1. Let the $U = \{1, 2, 3, \ldots, 10\}$ be a universal set and $A = \{1, 4, 7, 10\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{2, 4, 6, 8\}$. List the elements of each following set:
   (a) $\overline{A}$
   (b) $B \cap C$
   (c) $\overline{B} \cap (C \backslash A)$
   (d) $(A \cup B) \backslash (C \backslash B)$
2. Determine the cardinality of the following sets:
   (a) $\emptyset$
   (b) $\{\emptyset\}$
   (c) $\{a, bc, d\}$
   (d) $\{a, bc, d, \{a, bc, d\}\}$
3. For the following questions determine if $A \subseteq B$ or not.
   (a) $A = \{1, 2\}$, $B = \{3, 2, 1\}$
   (b) $A = \{1, 2\}$, $B = \{x : x^3 - 6x^2 + 11x = 6\}$
   (c) $A = \{x : x^3 - 2x^2 - x + 2 = 0\}$, $B = \{x^3 - 6x^2 + 11x - 6 = 0\}$
   (d) $A = \{1, 2, 3, 4\}$, $C = \{5, 6, 7, 8\}$, $B = \{n : n \in A \text{ and } n + m = 8 \text{ for some } m \in C\}$
4. For the following questions represent the following proposition symbolically using the following symbols:

   p: There is a hurricane

   q: It is raining

   (a) There is no hurricane
   (b) There is a hurricane, but it is not raining.
   (c) Either there is a hurricane or it is raining but there is no hurricane.
5. Determine the truth value of the following propositions:
   (a) If $3 + 5 < 2$ then $1 + 3 = 5$
   (b) $3 + 5 > 2$ if and only if $1 + 3 = 4$.
6. Using the following statements write the following propositions in symbols:

   p: You run 10 laps daily

   q: You are healthy

   r: You take multi-vitamins

   (a) If you run 10 laps daily, then you will be healthy.
   (b) Taking multi-vitamins is sufficient for being healthy
   (c) If you are healthy, then you run 10 laps daily or you do not take multi-vitamins.
7. Given that $P(x)$ denotes "$x$ is an accountant" and $Q(x)$ denotes "$x$ owns a Porsche" write each statement symbolically.
   (a) All accountants own Porsches

(b) Some accountant owns a Porsche

(c) Someone who owns a Porsche is an accountant.

8. Let $T(x, y)$ stand for the propositional function $x$ is taller than or the same height as $y$. Write all of the following propositions as words:
   (a) $\forall x \, \forall y \, T(x, y)$
   (b) $\forall x \, \exists y \, T(x, y)$
   (c) $\exists x \, \exists y \, T(x, y)$
   (d) $\exists x \, \forall y \, T(x, y)$

9. Prove the following claim, or provide counter examples:
   (a) For all integers $m$ and $n$, if $m$ and $m + n$ are even then $n$ is even.
   (b) For sets $X$, $Y$, and $Z$, if $X \subseteq Y$ then $Z \backslash Y \subseteq Z \backslash X$.
   (c) For sets $X$, $Y$, and $Z$, $X \times (Y \backslash Z) = (X \backslash Y) \times (X \backslash Z)$.
   (d) $\forall x \in \mathbb{R}$ if $x^2$ is irrational then $x$ is irrational.
   (e) $\sqrt[3]{2}$ is irrational.
   (f) For all $x, y \in \mathbb{R}$ if $x$ is rational and $y$ is irrational then $xy$ is irrational.
   (g) $(X \backslash Y) \cap (Y \backslash X) = \emptyset$ for all set $X$ and $Y$.
   (h) Let $s_1, s_2, \ldots, s_n$ be any real numbers. We define the average of these numbers as:

   $$A = \frac{s_1 + s_2 + \cdots + s_n}{n}$$

   Suppose there exists an $i$ and a $j$ such that $s_i \neq s_j$ then there must be some $k$ such that $s_k > A$.
   (i) For all sets $A$, $B$, and $C$, $A \subseteq C$ and $B \subseteq C$ if and only if $A \cup B \subseteq C$.
   (j) For all positive integers $n$ the following holds:

   $$\frac{1}{2n} \leq \frac{1 \cdot 3 \cdot 5 \cdots (2n - 1)}{2 \cdot 4 \cdot 6 \cdots (2n)}$$

   (k) If $X_1, X_2, \ldots, X_n$ and $X$ are sets then:

   $$X \cap (X_1 \cup X_2 \cup \cdots \cup X_n) = (X \cap X_1) \cup (X \cap X_2) \cup \cdots \cup (X \cap X_n)$$

   (l) If $X_1, X_2, \ldots, X_n$ and $X$ are sets then:

   $$\overline{X_1 \cap X_2 \cap \cdots \cap X_n} = \overline{X_1} \cup \overline{X_2} \cup \cdots \cup \overline{X_n}$$

10. Let $f$ and $g$ be functions from the positive integers to the positive integers defined by

    $$f(n) = 2n + 1, \qquad g(n) = 3n - 1$$

    Find the compositions $f \circ f$, $g \circ g$, $f \circ g$ and $g \circ f$.

11. Each of the following functions is one-to-one and thus is a bijective function from its domain to its image. Find the inverse of the given functions.
    (a) $f(x) = 4x + 2$, $x \in \mathbb{R}$.
    (b) $f(x) = 3 \log_2(x)$, $x \in \mathbb{R}^{>0}$.

(c) $f(x) = 6 + 2^{7x-1}$, $x \in \mathbb{R}$.

12. Let $f$ be the function from $X = \{0, 1, 2, 3, 4, 5\}$ to $X$ defined by

$$f(x) = 4x \mod 6$$

Write $f$ as a set of ordered pairs and draw the arrow diagram for $f$. Is $f$ injective? Surjective?

13. For the following questions let $g$ be a function from $X$ to $Y$ and let $f$ be a function from $Y$ to $Z$. For each of the following if the statement is true prove it otherwise proved a counterexample.
    (a) If $g$ is onto then $f \circ g$ is onto.
    (b) If $f \circ g$ is injective then $f$ is injective.
    (c) If $f$ is one-to-one then $f \circ g$ is one-to-one.