CSCI 2824 - Discrete Structures
Test 1

Grade: _____ /100

1. (10 points) For the following pair of propositions show whether P is logically equivalent to Q:

   (a) $P = p \rightarrow q$, $Q = \neg p \vee q$

   (b) $P = p \wedge q$, $Q = \neg q \rightarrow p$.

2. (20 points) For the following parts list each set that has the same cardinality as the given set, from the provided list of sets.

   $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{Z} \times \mathbb{Z}$, $\{1, 3\}$, $\{a, 7\}$, $\{1, 2, 4, 5\}$, $\{1, 2, 3, 4, 5\}$, $\{1, \{2, 3\}, \{4\}\}$, $\{7, 132, 19, \{1\}\}$

   (a) E - the set of all even integers.

(b) $[0, 1]$ - all real numbers between 0 and 1, inclusive.

(c) $\{a, b\}$

(d) $\{7, 24, 5\}$

3. (10 points) Let $A(x, y)$ be the propositional function x attended y's office hours and $E(x)$ be the propositional function x is enrolled in a discrete mathematics course. Using these and our quantifiers turn the following statements into symbols that they are equivalent to.

(a) Brit attended someone's office hours.

(b) Every discrete math student attended someone's office hours.

4. (20 points) Prove that $\mathcal{P}(X \cap Y) = \mathcal{P}(X) \cap \mathcal{P}(Y)$ for all sets X and Y.

5. (25 points) Prove that $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

6. (5 points) Define what it means that $f : X \to Y$ is a function.

7. (10 points) For functions $f : Y \to Z$ and $g : X \to Y$ prove or disprove the following claim: If $f \circ g$ is onto then g is onto.