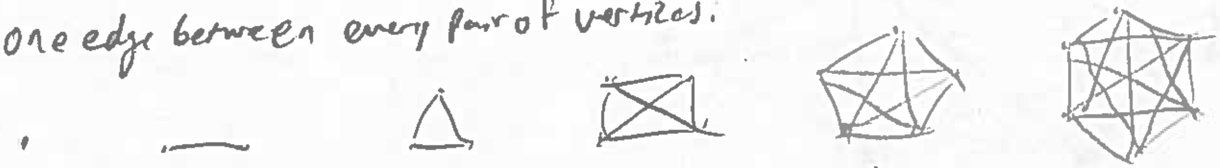
Special graphs:

Ex: Complete graphs: A <u>complete graph</u> on n vertices K_n ~~has~~ contains one edge between every pair of vertices.



K_1 K_2 K_3 K_4 K_5 K_6

How many edges does K_n have? $\deg(v) = ?$ $n-1$

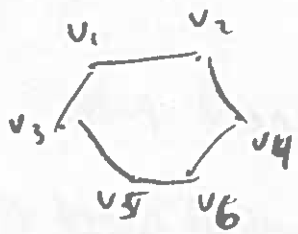
$$\Rightarrow \sum_{v \in V} \deg(v) = n \cdot (n-1) \Rightarrow \#\text{ of edges} = \frac{n(n-1)}{2}$$

or given n ~~edges~~ vertices how many pairs of vertices are there? $\binom{n}{2} = \frac{n(n-1)}{2}$

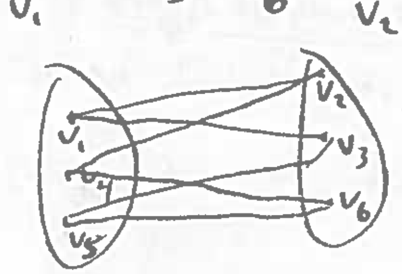
Bipartite Graphs:

Def: A simple graph G is <u>bipartite</u> if V can be partitioned into 2 disjoint sets V_1 & V_2 s.t. that every edge only goes between nodes in V_1 & V_2.

Ex

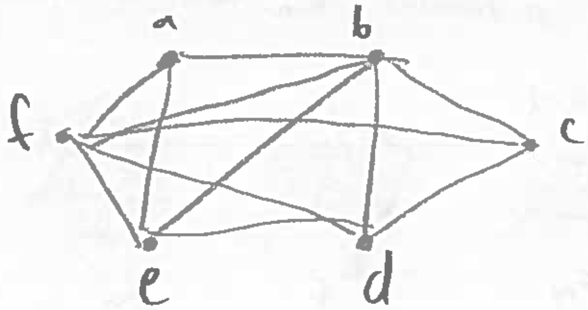


is bipartite



no edges within set.

Ex: Is G bipartite?



No, Consider a, b, f. a, b, f cannot be in the same group since $(f,a) \in E$, (f,b), (a,b) but since there are only two groups two of them must be in the same set.

<u>Theorem</u>: A simple graph is bipartite iff it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices have the same color.

That exactly determines the sets.

This is called 2-coloring a graph.

<u>Connectivity</u>:

We wish to discuss how connected graphs are. First we need paths.

<u>Def</u>: Let $n \in \mathbb{N}$ and G an undirected graph. A <u>path of length n</u> from u to v is a sequence of edges $e_1, \dots, e_n$ in G for which there exist a sequence of nodes $x_0 = u, x_1, \dots, x_n = v$ where $e_i = (x_{i-1}, x_i)$

The path is a <u>circuit</u> if it begins and ends at the same vertex.

A path is <u>simple</u> if it does not repeat edges