Similarly $\neg(\exists x\, Q(x)) \equiv \forall x \neg Q(x)$

if there does not exist an x s.t. $Q(x)$ then every x gives $\neg Q(x)$.

Ex: Negate the sentence "Some one will fail this class" where $P(x)$ = pass class

Some one will fail this class $\Rightarrow \exists x \neg P(x)$

$\neg(\exists x \neg P(x)) \equiv \forall x \neg\neg \cancel{P}(x) \equiv \forall x\, P(x)$

"Every one will pass this class."

Now onto fun stuff!

First, puzzles: Suppose there are two people and they either only tell the truth or only lie, independently. A telling the truth has no implication on B. They then say:

A: "Exactly one of us is lying"

B: "At least one of us is truthful"

What can you conclude about the truthfulness of A/B?

Lets do this rigorously:

A truthteller?	B truthteller?	A told truth?	~~A~~B told truth?	Viable
T	T	F	T	NO.
T	F	T	T	NO,
F	T	T	T	NO
F	F	F	F	Yes,

Puzzle: Four prisoners are arrested for a crime. However the jail is full. So the jailor makes them a deal, He will give them a puzzle, if they solve it they go free, otherwise they are executed.

The puzzle: Three men are put in a line B facing a wall, C facing B D facing C, D can see C & B. A is placed behind a curtain & can see no one & no one can see him.

A | D C B |

If any prisoner moves or turns around they are all executed. Each prisoner is given a hat, they are told there are 2 white & 2 black hats. If any prisoner can deduce the color of their own hat (with no communication) they are all set free. If a wrong answer is given they all die. They have no prep time for a strategy.

How do they get set free?

Class Demo!

A can't be seen or see anything so he should stay quiet.

B can't see anything so he should be quiet.

If D sees C & B have the same color then he knows his own color.

If D doesn't say anything C knows he has a different color than B so he can guess his color

Puzzle:

You are a great logician. The gov't comes to you & says that they have been given a counterfeit quarter but need your help identifying it. They have 12 quarters (one of which is fake) & a scale. You know the counterfeit will weigh either less or more than normal.

The gov't official becomes upset when you say you must weigh them all. he wants it done immediately. He allows you to use the scale 3 times, if you don't have a solution, he'll arrest you for counterfeiting. How do you find the counterfeit coin?

Solution: Split the coins into 3 groups of 4. Weigh two groups against each other. Two cases:

1. Sides balance ⇒ all 8 on scale are legit. Mark those w/ a O

   Now take 3 balanced coins & weigh against 3 unmarked coins.
   Two cases:

   1a. They balance ⇒ remaining coin is counterfeit (2 weighings)

   1b. Imbalance ⇒ Draw a +/− on coins if they are heavier/lighter

      Take two +/−'d coins & weigh against each other
      Two cases:

      1ba. They balance ⇒ remaining coin is counterfeit (3 weighings)

      ~~b~~1bb. Imbalance ⇒ If they are + ⇒ heavier coin is fake
      If they are − ⇒ lighter coin is fake
      (3 weighings)

2. Imbalance ⇒ Mark heavier coins with + lighter with − & remaining w/ O

   replace 3 heavier coins w 3 lighter coins. 3 lighter coins replaced with 3 O coins. Three possibilities.

2a. If heavier side is still heavier ⇒ remaining heavy coin is fake or
remaining light coin is fake

Choose one & weigh against one of the known fake ones
Two cases:

2a a: Scales balance ⇒ un chosen coin is fake (3 weighings)

2a b: Imbalance ⇒ chosen coin is fake (3 weighings)

2b. If heavier side is now lighter ⇒ one of 3 light coins is fake.

Choose 2 of them & weigh against each other. Two cases:

2b a. Scales balance ⇒ 3rd light coin is fake (3 weighings)

2b b. Imbalance ⇒ lighter side is fake (3. weighings)

2c. Now sides balance ⇒ One of 3 removed heavy coins is fake

Choose 2 & weigh against each other. Two cases:

2c a. Scales balance ⇒ 3rd heavier coin is fake (3 weighings)

2c b. Imbalance ⇒ heavier coin is fake (3 weighings)

<u>Now to paradoxes!</u>

Paradoxes are statements that <u>cannot</u> have logical assignments.

<u>Ex</u>: This sentence is False.

If that's a <u>true</u> sentence then it must be <u>False</u>

If that's a <u>false</u> sentence then it was <u>truthful</u>.

Lottery paradox: Consider a fair 1000 ticket lottery, with exactly one winning ticket. It is rational to assume there is 1 winning ticket. Given the supposition that an event will not occur if the probability is less than 0.002 we can then rationally conclude ticket 1 will not win, similarly 2, 3, ... 1000 will not win. Thus we can conclude that no ticket will win. $\rightarrow\leftarrow$??

Less a logical paradox more misunderstand of probability, but still fun.

Now my favorite: The Barber paradox: Suppose there is a town where every man is clean shaven. The town has a barber shop which has one barber. For every man in the town there is a rule: the barber will shave those who do not shave themselves (and no one else).

Does the barber shave himself?

If he shaves himself then he may not shave himself. If he does not shave himself then he must shave himself.

Unexpected Hanging: A prisoner is convicted and sentenced to hanging. His judge tells him he will be hanged at noon one week day in the following week & the execution will be a surprise. He will not know the day until the executioner knocks on his door at noon.

The prisoner then concludes he will escape from his hanging as follows:
His "surprise hanging" can't be on Friday, if he isn't hanged on Thursday he knows it will be Friday $\Rightarrow$ no surprise.

This means he also can't be hanged on Thursday, if it doesn't happen Wednesday it has to happen Thursday $\Rightarrow$ no surprise. He concludes he cannot be hanged.

The next week the executioner knocks on his door on Wednesday & hangs him.
Despite everything he was surprised.