Propositional Logic:

Logic allows us to be rigorous & precise about our claims. Besides providing a background language for mathematical claims & proofs it is the backbone of computer science. Used in the design of computer circuits, construction of programs, verification of correctness.

Def: A proposition is a declarative sentence (statement) that is either true or false but not both.

Ex: 1. Washington D.C. is the Capital of U.S.A.

2. Toronto is the Capital of Canada.

3. $1+1=2$.

4. $2+2=3$.

5. Earth is the only planet containing life.

1, 3 True, 2, 4 False 5 unknown, but is either true or false

Exs. 1. Buy two tickets to the Blink 182 Concert.

2. $x+4=6$

3. The difference of two primes.

1 & 3 are not declaratives (3 is a fragment)

2. Neither true nor false, or value varies based on x.

Exs: 1. $2+5=19$

2. $x+9=15$

3. Every even integer greater than 4 is the sum of two primes.

4. For some positive integer n $19340 = n \cdot 17$

Let class answer : 1 ✓ (F)

2 X

3 ✓ (unproven Goldbach's Conjecture)

4 ✓ (F)

Often we use propositional variables (p, q, r, s) to represent propositions. & use T/F for true, false.

e.g. p: $2+5=19$ proposition p is F.

Def: Let p be a proposition. The negation of p, denoted by $\neg p$, $\overline{p}$, or $\sim p$ is the statement It is not the case that p.

$\neg p$ is read "not p" the truth value of $\neg p$ is the opposite of p.

Ex: Find the negation of
p: You play football

sol: $\neg p$: It is not the case that you play football

or: You do not play football.

Ex: Negate the following

q: you run at least 10 laps daily.

$\neg q$: It is not the case you run at least 10 laps daily.

or: You do not run at least 10 laps daily

or: you run less than 10 laps daily.

We can show truth values in Truth tables

This is to show the effect of an operation - independent of the actual proposition

Ex: truth table for negation

p	$\neg p$
T	F
F	T

tells us if $p = T$ then $\neg p = F$ etc.

A truth table must show all possible options.

Def: Let p & q be propositions. The Conjunction of p & q, denoted $p \wedge q$ is the proposition p and q.

The Conjunction $p \wedge q$ is true when p & q are both true & false otherwise.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

← Note need 4 rows for every option between p & q

Ex: Find the conjunction $p \wedge q$ for p & q as above

p: You play football

q: you run at least 10 laps daily

$p \wedge q$: you play football & run at least 10 laps daily.

Def: Let p & q be propositions. The disjunction of p & q, denoted $p \vee q$ is the proposition p or q.

The disjunction $p \vee q$ is false if p & q are both false, & true otherwise

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The disjunction means inclusive or

at least (and possibly both) are true.

Ex: 6 is even or a multiple of 3.

Some statements are exclusive or only one can be true

Ex: You will pass this course or be grounded

This is usually denoted $p \oplus q$.

let students fill this out

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statements:

Def: Let p, q be propositions. The Conditional Statement $p \to q$ is the proposition If p then q.

$p \to q$ is false if p is true but q is false. And true otherwise.

p is called the hypothesis & q the conclusion.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Think of this as a promise if you do p then I'll do q. if you don't do p then no matter what I do I've lived up to the promise.

There are many ways to write $p \to q$

If p then q, p implies q, p is sufficient for q.

Ex: Express $p \to q$ in english for

p: The CS department gets an additional \$60,000

q: The CS department will hire one new faculty.

$p \to q$: If the CS department gets an additional \$60,000 then it will hire one new faculty.

Ex: Consider If Juan has a smart phone then $2+3=5$

Is this true?

Yes! the Conclusion is a true statement so the hypothesis is irrelevant the Whole Conditional is true!

We can form new conditionals from $p \to q$

Def: The proposition $q \to p$ is the Converse of $p \to q$

The proposition $\neg q \to \neg p$ is the Contrapositive of $p \to q$

The proposition $\neg p \to \neg q$ is the inverse of $p \to q$

Note We can prove $p \to q$ is the Same as $\neg q \to \neg p$

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Two propositions are equivalent if they have exactly the same truth values.

Def: Let p & q be propositions. The <u>biconditional statement</u> $p \leftrightarrow q$ is the proposition p if and only if q. The biconditional statement $p \leftrightarrow q$ is true whenever p & q have the same truth values & false otherwise.

Note $p \leftrightarrow q$ is true when both $p \to q$ & $q \to p$ are true.

if and only if is often abreviated iff.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Ex: Construct the truth table of the compound proposition

$$(p \wedge q) \to (\neg p \vee q)$$

p	q	$\neg p$	$p \wedge q$	$\neg p \vee q$	$(p \wedge q) \to (\neg p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	F	T	T
F	F	T	F	T	T

← Tautology always true.

Applications of logic

One major application of logic is Boolean searching.

For instance looking for "new mexico universities"

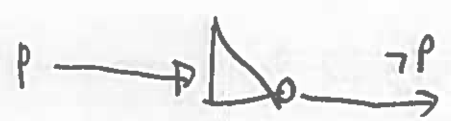
you can search web pages for "new" AND "Mexico" AND "universities"

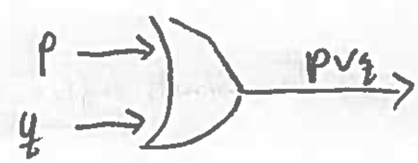
when you probably meant "new Mexico" AND "universities"

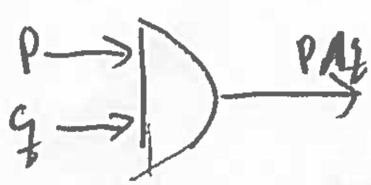
so search engines must derive "meaning" from words.

Logic Puzzles: Wednesday :)

Logic Circuits in Computers: we use gates:







Ex: What is the output if: