Permutations & Combinations

Many Counting Problems Can be solved by determining the number of ways we can order objects

<u>Ex</u>: In how many ways can we select 3 Students from a group of 5 students to stand in line for a Picture?

Note the order we choose students matters! ABD is different than BAD. We can use our product rule to solve this:

$\underline{5} \cdot \underline{4} \cdot \underline{3} = 60$ ways.

<u>Def</u>: A <u>Permutation</u> of a set of distinct objects is an ordered arrangement of these objects.

<u>Ex</u>: 3,1,2 is a permutation of $\{1,2,3\}$ 3,1 is a 2-permutation of $\{1,2,3\}$

<u>Theorem</u>: Let $n \in \mathbb{N}$ $r \in \mathbb{N}$ with $1 \le r \le n$. Then there are

$$P(n,r) = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

r-permutations on a set of n elements.

<u>Ex</u>: How many permutations of A, B, C, D, E, F are there?

6!

How many if DEF must be consecutive (in that order)?

Now we have 4 tokens A, B, C, DEF so 4!

How many if DEF must be together in any order?

Same problem as before, but now we can re order each DEF.

Take each of the ~~24~~ 24 solutions before & rearrange DEF.

How many ways to re arrange DEF? $3! = 6$

So total $= 24 \cdot 6 = 144$.

<u>Ex</u>: How many ways can six people be seated around a circular table? Given that any rotation is the same seating

i.e.

```
     A                  F
  F     B            E     A
               =             B
  E     C             D  C
     D
```

<u>Sol</u>: Choose one person & seat them arbitrarily. So seat A at the top

```
     A
  _     _
  _     _
     _
```

Now we just need to seat the five people, only order matters! $\Rightarrow$ Permutation $\Rightarrow$ $5!$ ways

Alternatively 6 spaces so $6!$ but every solution has 6 equal answers so $\frac{6!}{6} = 5!$.