then by construction r is not on our list but is between 0 & 1.

notice r disagrees w/ r_i at spot r_{ii} so $r \neq r_{ii}$ $\forall i \in \mathbb{N}$

Thus $(0,1)$ is not countable $\Rightarrow$ $\mathbb{R}$ is not countable $\Rightarrow$ $\mathbb{R}$ is uncountable.

$\square$.

Note by convention $|\mathbb{R}| = |P(\mathbb{N})|$

This proof secretly relies on $1 = 0.999\ldots$

Because we need every real number to be distinct from others,

So lets prove this LOTS of WORK, Ready ?!

Ex: $1 = 0.999$

Pf Let $x = 0.999\ldots$

$10x = 9.999\ldots$

$= 9 + 0.9999\ldots$ or

$= 9 + x$

$9x = 9$

$x = 1$

$\frac{1}{9} = 0.11111\ldots$

$\Rightarrow 1 = \frac{9}{9} = 0.99999$

There are analytic proofs as well that demonstrate there is no number between 1 & $0.999\ldots$

Matrices:

Def: A matrix is a rectangular array of numbers. A matrix with m rows & n columns is called an $m \times n$ matrix. If $m = n$ then the matrix is square.

Ex: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ is a 3×2 matrix.

We often write Matrices more generally as $\{a_{ij}\}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & & a_{2n} \\ \vdots & & & & \\ a_{m1} & \cdots & & & a_{mn} \end{bmatrix}$$

We can add matrices, when they are the same size:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 7 \\ 0 & 6 \end{bmatrix}$$

We can multiply two matrices in a very particular case.

If $A = n \times m$ & $B = p \times k$ then we can multiply A & B iff $m = p$. Then $A \times B = n \times k$ in size. It is computed as follows:

$C = \{c_{ij}\}$ where $c_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \cdots + a_{im} b_{mj}$

Ex: $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 5 & 6 \end{bmatrix}_{3 \times 2}$ $B = \begin{bmatrix} 5 & 5 & 5 & 5 \\ 6 & 6 & 6 & 6 \end{bmatrix}_{2 \times 4}$

$\Rightarrow C = 3 \times 4$ $c_{11} = 1 \cdot 5 + 3 \cdot 6 = 23$ $c_{12} = 1 \cdot 5 + 3 \cdot 6 = 23$

$$C = \begin{bmatrix} 23 & 23 & 23 & 23 \\ 34 & 34 & 34 & 34 \\ 61 & 61 & 61 & 61 \end{bmatrix}$$

Can think of this as taking a row of A & multiply it by a column of B.

Note $AB \neq BA$ in general. In fact in our example BA doesn't exist.

Ex: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$AB = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$ $BA = \begin{bmatrix} 23 & 34 \\ 31 & 46 \end{bmatrix}$

$AB \neq BA$.

Def: The matrix $\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}_{n \times n}$ is the identity matrix I_n

e.g. $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ etc.

If A is a k square matrix we can define $A^n = \underbrace{A \cdot A \cdot A \cdots A}_{n \text{ times}}$

if $n = 0$ $A^0 = I_k$

We can flip matrices: If $A = k \times n$ then $A^T = n \times k$ is the transpose of A

$A = \{c_{ij}\}$ $A^T = \{c_{ji}\}$

Ex: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

rows become columns & columns become rows.