Introduction to Counting

Now we're going to learn to count!

Specifically, we'll count the number of possible outcomes.

First we'll need some rules:

The product rule: If a procedure can be broken into two (or more) tasks & there are n_i ways to do the ith task there are $n_1 n_2 n_3 \cdots n_k$ ways to do the procedure.

Ex: If a menu consists of the following main courses & drinks:

Hamburger	Tea
Cheeseburger	Milk
Fish Filet	Soda
	Coffee

How many ways are there to have a meal, which consists of a main course & optionally a drink?

3 main courses   5 drinks (one no-drink)

= 15 possible meals.

Ex: How many strings of length 4 can be formed using the letters A B C D E if repetitions are allowed? Not allowed? How many begin with B? (no reps)

(b) $\underline{5}\ \underline{4}\ \underline{3}\ \underline{2} = 5! = 120$ (a) $\underline{5}\ \underline{5}\ \underline{5}\ \underline{5} = 5^6$

1st letter 2nd 3rd 4th

# of choices for each letter

(c) $\underline{1}\ \underline{4}\ \underline{3}\ \underline{2} = 4! = 24$

<u>Ex</u> How many possible Random Colorado license plates are there?

3 letters then 3 numbers or 3 numbers then 3 letters.

$\underline{26}\,\underline{26}\,\underline{26}\quad \underline{10}\,\underline{10}\,\underline{10}$ $\qquad$ $\underline{10}\,\underline{10}\,\underline{10}\quad \underline{26}\,\underline{26}\,\underline{26}$

$$= 26^3 \cdot 10^3 \cdot 2$$

<u>Ex</u>: How many functions are there from a set of m elements to a set with n elements?

$$\underbrace{\underline{n}\ \underline{n}\ \underline{n}\ \underline{n}\ \cdots\ \underline{n}\ \underline{n}}_{m \text{ inputs}}$$

<u>Ex</u>: How many 1-to-1 fcns? $\qquad$ If $m > n$ then 0 if $m \le n$

$$\underbrace{\underline{n}\ \underline{n-1}\ \underline{n-2}\ \underline{n-3}\ \underline{n-4}\ \cdots\ \underline{n-m+1}}_{m} = \frac{n!}{m!}$$

<u>Sum Rule</u>: If a task can be done in either n_1 ways or n_2 ways & none of the n_1 or n_2 ways are the same Then there are $n_1 + n_2$ ways to do the task.

<u>Ex</u>: In how many ways can we select two books from different subjects among 5 distinct CS Books, 3 distinct math books, 2 distinct art books?

Prod rule says we can select a CS & Math book in 15 ways

a CS & art book in 10 ways

a Math & art book in 6 ways

Sum rule says two books in different subjects in $15 + 10 + 6 = 31$ ways.

Ex: A six person committee composed of Alice, Bob, Charlie, Dylan, Elle, & Frank is to select a chairperson, secretary, treasury from themselves.

(a) How many ways can this be done?

(b) If either Alice or Bob must be chairperson?

(c) If Elle must hold one position?

(d) If Both Dylan & Frank must hold positions?

(a) $6 \cdot 5 \cdot 4 = 120$ product rule

(b) $2 \cdot 5 \cdot 4 = 20$ product rule

(c) $1 \cdot 5 \cdot 4 + 5 \cdot 1 \cdot 4 + 5 \cdot 4 \cdot 1 = 3 \cdot 20$ (sum & product rules)

(d) More complicated. we have many tasks here

1. find a role for Dylan
2. find a role for Frank
3. Fill remaining position

1. 3 options
2. 2 options (1 taken by Dylan)
3. 4 options (other 4 fill remaining position)

$= 3 \cdot 2 \cdot 4 = 24$

Ex: In the programming language BASIC (older version) the name of variables needed to be two characters (letter or number) uppercase was not distinguished from lower case. Variables had to start with letters & could not be one of the 5 reserved 2 letter words. How many possible variable names were there?

e.g. a5 ✓ A5 = a5 if X abcX 5SX a ✓ 5X

How many 1 character ones? 26

How many 2 characters? $\underline{26}\ \underline{36}$ but 5 are reserved

$26 \cdot 36 - 5$

Altogether $26 + 26 \cdot 36 - 5 = 957$ options.

Ex: passwords: How many possible passwords are there of length 8 allowing letters, upper & lower numbers & symbols

32

total number of options $32 + 10 + 26 + 26 = 94$

So $\underline{94}\ \underline{94}\ \underline{94}\ \underline{94}\ \underline{94}\ \underline{94}\ \underline{94}\ \underline{94} = 94^8 = 6095689385410816$

What if one symbol, one upper, one lower case, & one number are required?

Many tasks:
1. place symbol → 8
2. place upper → 7
3. place lower → 6
4. place number → 5
5. fill remaining 4 spaces → 94^4

$8 \cdot 7 \cdot 6 \cdot 5 \cdot 94^4$

$= 131165825280$

2×10^{-7} % of possible 8 length passwords!

password restrictions might make dictionary searches more difficult

but allow brute force searches! (we keepass!).

Ex: How many bit strings of length 8 either start with 1 or end with 00 but not both?

Need the inclusion - exclusion rule - purposely over count then count how much you over count:

Start 1: $\underline{1}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2} = 2^7$ $\quad = 2^7 + 2^6 - 2^5 = 160$ ways.

end with 00: $\underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{1}\ \underline{1} = 2^6$

Both: $\underline{1}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{1}\ \underline{1} = 2^5$