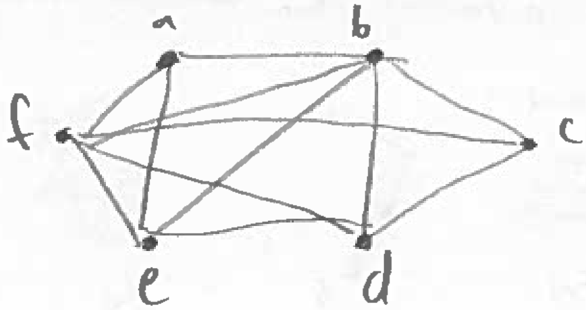
Ex: Is G bipartite ?



No, Consider a, b, f   a, b, f cannot be in the same group since $(f,a) \in E \ni (f,b)$, (a,b) but since there are only two groups two of them must be in the same set.

Theorem: A simple graph is bipartite iff it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices have the same color.

That exactly determines the sets.

This is called 2-Coloring a graph.

Connectivity:

We wish to discuss how connected graphs are. First we need paths.

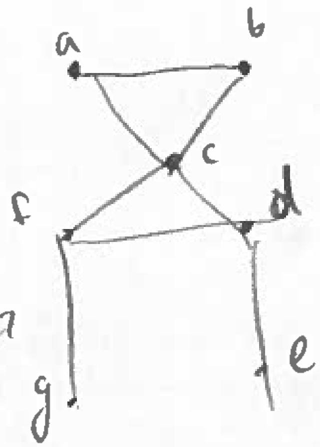
Def: Let $n \in \mathbb{N}$ and G an undirected graph. A path of length n from u to v is a sequence of edges $e_1, \dots, e_n$ in G for which exist a sequence of nodes $x_0 = u, x_1, \dots, x_n = v$ where $e_i = (x_{i-1}, x_i)$

The path is a circuit if it begins and ends at the same vertex.

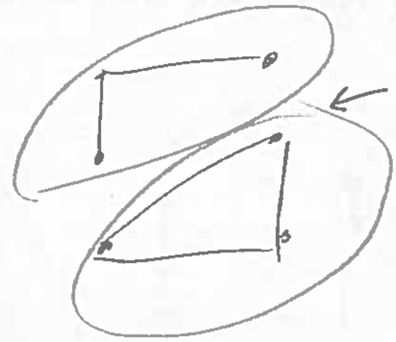
A path is simple if it does not repeat edges

Def: An undirected graph G is called Connected if there is a path between any two distinct nodes. A graph that is not Connected is disconnected.

Ex:



← Connected



← not Connected.

↑ Connected Components

Theorem: There is a (simple) path between every pair of distinct vertices of a connected undirected graph.

Def: A connected component of G is a subgraph that is connected. (Must be maximal)

E.g. whole thing is connected component, a part of it is not.

Imagining a graph as a computer network it being connected tells us that any two computers can communicate with each other.

A standard network question, how reliable is it? What if one computer/router goes out can all comps still communicate?

we need to answer how connected a graph is.

Def: A cut vertex is a vertex in a connected graph whose removal creates more connected components.

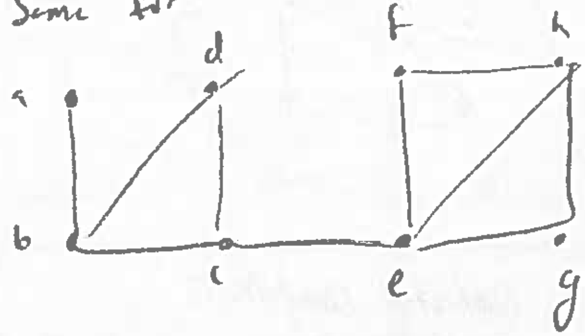
Analogously there is the notion of a cut edge.

Ex: Find all cut vertices & edges in previous example.

c, f, d are cut vertices

(d,e) (f,g) are the cut edges.

Ex: Same for



b, C, e are cut vertices

(a,b), (c,e), are cut edges.

In directed graph connectedness is the same except paths must follow direction of edges.