Combinations.

Now we solve problems with no ordering!

Ex: How many committees of 3 ppl can we form from a group of 4 students?

Notice in this case the committee of A, B, C is the same as C, A, B.

So we're asking how many 3-subsets of there of a set w/ 4 elements.

The answer is 4 $\{A,B,C\}$ $\{A,B,D\}$, $\{A,C,D\}$, $\{B,C,D\}$.

more common

This is called an r-combination this is denoted $C(n,r)$ or $\binom{n}{r}$

& is read n choose r.

Theorem: The number of r-combinations of a set of n elements where $n, r \in \mathbb{N}$ $0 \le r \le n$ equals

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

note $\frac{n!}{(n-r)!} = P(n,r)$ number of ways to order r elements from n divide by r! Since that is the number of ways to order r elements (from r) all of which are the same!

Ex: In how many ways can we select a committee of 2 women & 3 men from a group of 5 women & 6 men?

First choose the 2 women $\binom{5}{2} = \frac{5!}{2!\,3!} = \frac{5 \cdot 4}{2 \cdot 1} = 10$

then choose men $\binom{6}{3} = \frac{6!}{3!\,3!} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$

Thus there are $10 \cdot 20 = 200$ ways to make this committee

Ex: How many 5 card poker hands are there from a standard 52-card deck?

$$\binom{52}{5} = 2{,}598{,}960$$

How many contain the same suit?

$$\binom{4}{1}\binom{13}{5} = \frac{4!}{3!\,1!} \cdot \frac{13!}{5!\,(8!)} = 4 \cdot \frac{13 \cdot \cancel{12} \cdot 11 \cdot \cancel{10} \cdot 9}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$$

choose suit → $\binom{4}{1}$; choose 5 cards → $\binom{13}{5}$

$$= 4 \cdot 13 \cdot 11 \cdot 9 = 5148$$

How many contain 3 cards the same value & 2 a second value?

$$\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2} = 3744$$

choose the first value → $\binom{13}{1}$; choose the suits → $\binom{4}{3}$; choose second suit → $\binom{12}{1}$; choose suits → $\binom{4}{2}$

Ex: How many bit strings of length 8 contain at least 5 0s?

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Choose our 5 0 bits $\binom{8}{5}$ then last 3 can contain anything 2^3 options

$\Rightarrow \binom{8}{5} 2^3$ options.

wrong! You've double counted! e.g.

00000 _ _ _		1
0 1 0		11
_ 0 000 0 _ _		2
0	1 0	

Correct way? $\binom{8}{5} + \binom{8}{6} + \binom{8}{7} + \binom{8}{8}$