CSCI 2824 - Discrete Structures
Homework 1

You MUST show your work. If you only present answers you will receive minimal credit. This homework is worth 100pts with 5 bonus points.

Due: Wednesday June 14

1. (4 points) Determine if the following are propositions or not:
   (a) $2 + 5 = 19$
   (b) $x + 4 > 10$
   (c) Audrey Meadows was the original "Alice" in "The Honey-Mooners".
   (d) This statement is false.

2. (9 points) Find the bitwise OR, AND, and XOR of the following pairs of bit strings.
   (a) 0101 1110, 0010 0001
   (b) 1111 0000, 1010 1010
   (c) 0000 0111 0001, 10 0100 1000

3. (15 points) Write the truth table for each of the following propositions:
   (a) $\neg(p \wedge q) \vee (r \wedge \neg p)$
   (b) $(p \vee q) \wedge \neg p$
   (c) $\neg(p \wedge q) \vee (\neg q \vee r)$

4. Given the following statements, and formulas de-construct the symbolic expressions into words.

   p: *Today is Monday*

   q: *It is raining*

   r: *It is hot*

   (a) (3 points) $\neg p \wedge (q \vee r)$
   (b) (7 points) $(p \wedge (q \vee r)) \wedge (r \vee (q \vee p))$

5. (12 points) There are 3 people, A, B, C. Each person either only tells the truth (is truthful) or only tells lies (is not truthful), independently (that is A being truthful does not imply B is or is not truthful, etc.). They then say the statements:

   A: "Exactly one of us is telling the truth."

   B: "We are all lying."

   C: "The other two are lying."

   What can you conclude about the identities about A, B, C? Are they liars or truthful? You should do this problem similar to the one in class with a truth-table considering each possibility and determining if it is viable.

6. (5 points (bonus)) This question is trickier, and more of a riddle than a puzzle. Two people, A and B are on an island and they either both tell the truth or both lie. There are also two paths on this island, one leads to certain death, the other to paradise. You are allowed to ask one question to determine your path, which you must then stick with. What question do you ask, that guarantees you go to paradise? Explain why it works.

7. (6 points) In class we talked about some of De Morgan's Laws, I proved some of them, you should prove the others:
   (a) Show that $\neg(p \vee q) \Leftrightarrow (\neg p) \wedge (\neg q)$, you should provide a truth table.
   (b) Show that $\neg(\exists x\ P(x)) \Leftrightarrow \forall x\ \neg P(x)$, you should provide an argument for this.

8. (10 points) Determine whether the following propositions are logically equivalent or not:
   (a) $p \rightarrow q$ and $\neg q \rightarrow \neg p$
   (b) $(p \rightarrow q) \wedge (q \rightarrow r)$ and $p \rightarrow r$

9. (6 points) Write the following propositions symbolically using quantifiers with the predicate $L(x, y)$ to mean x loves y. Do you think either are true?
   (a) Somebody loves everybody.
   (b) Somebody loves somebody.

10. (6 points) Let $P(x)$ denote x is a professional athlete and $Q(x)$ denote x plays soccer. For each of the following propositions determine their truth values and write the proposition in words.
    (a) $\forall x\ (P(x) \rightarrow Q(x))$
    (b) $\exists x\ (P(x) \vee Q(x))$

11. (9 points) Determine the truth value of each statement, where x, y are real numbers.
    (a) $\forall x\ \forall y\ (x^2 < y + 1)$
    (b) $\forall x\ \exists y\ (x^2 + y^2 = 9)$
    (c) $\forall x\ \exists y\ ((x < y) \rightarrow (x^2 < y^2))$

12. (4 points) Translate the following into symbolic logic using the given variables: "To use the wireless network in the airport you must pay the daily fee unless you are a subscriber to the service."
    - w: "You can use the wireless network in the airport"
    - d: "You pay the daily fee"
    - s: "You are a subscriber to the service"

13. (9 points) In a particular isolated tribal village of 100 people, all of which have blond hair, everyone follows a particular set of rules. Any person that knows their own hair color must leave the village forever. Thus every morning there is a ceremony where the elder asks if anyone knows their hair color, and if they do they are kicked out (and everyone else in the village instantly knows as well).

    Every person is completely truthful, and will follow this rule. But since no one wants to be kicked out of the village they go to great lengths to avoid seeing their reflection, and telling others what color their hair is.

    Eventually, an archaeologist stumbles upon this isolated village and walks among them for a time. The archaeologist mentions (when the whole village can hear her) "someone village member has blond hair" and then immediately leaves.

    What happens (assuming no more discussion on hair color occurs)?